Business Finance : course n°3
The Time Value of Money

BA 2nd Year - 01/10/2019

Time value of money

- The concept of time value of money is based on the notion that money has a « time value » because its value today is higher than the value tomorrow.

- Put it simply, $1 today is worth more than $1 in the future.

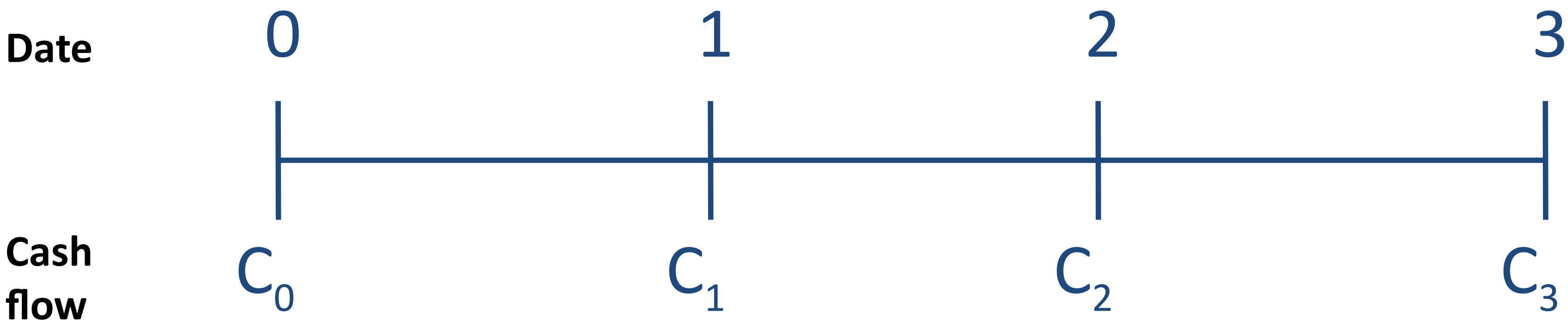
- Why is that so?

Time value of money

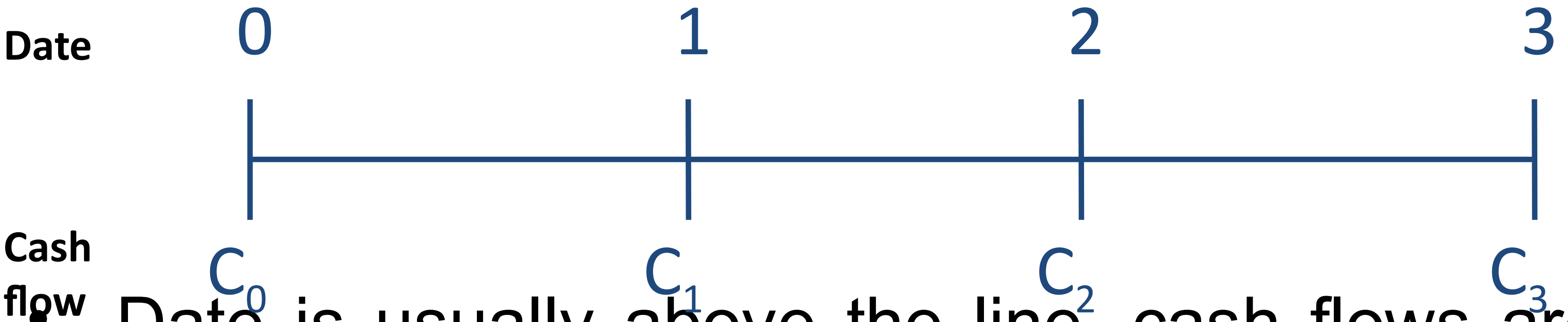
- $1 today can be put in the bank so that the value of the investment in the future will be more than $1 by the amount of interest earned.
- Therefore, the time value of money corresponds to the difference between money today and money in the future.

Timeline

- Timeline is a representation of the timing of expected future cash flows



Timeline



- Date is usually above the line, cash flows are presented below the line.
  - Date 0 is today (present), date 1 is one year later (end of the first year or beginning of the second year) etc…
  - Cash received (cash inflows) appears with a positive sign, cash paid (cash outflows) appears with a negative sign.

Timeline: examples

- Your parents have promised to offer you $2,000 one year from now and $2,000 two years from now.

- Show the timeline from your perspective.

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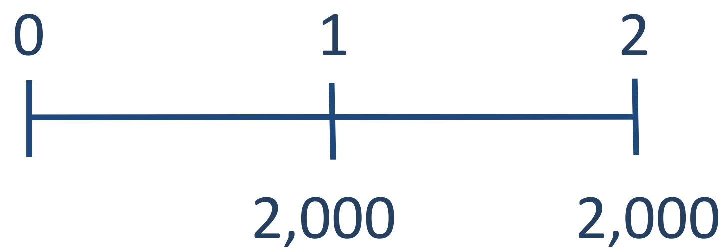
- Answer:



Timeline : examples

- Today, you have decided to take a four-year loan of $3,000. You will need to make annual payments of $1,000 at the end of each year.

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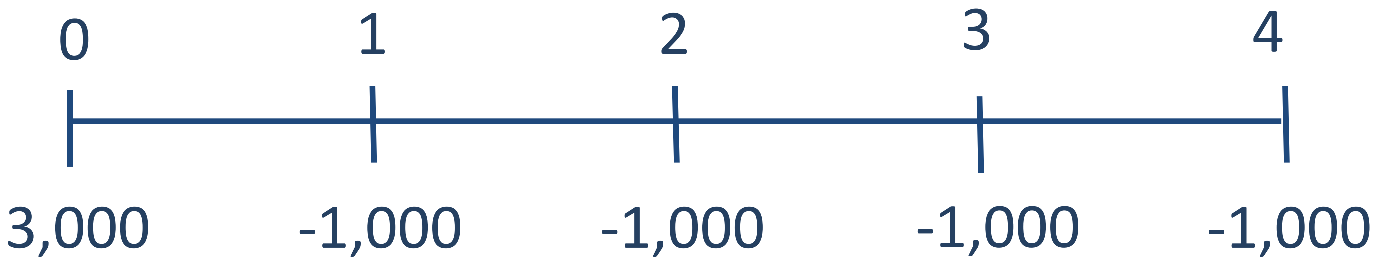


Time travel

- Financial decisions often require comparing or combining cash flows that occur at different points in time.

- The value of $1 today is higher than the value of $1 one year later because you can invest $1 today at the interest rate r and get $(1+r) at the end of the first year.

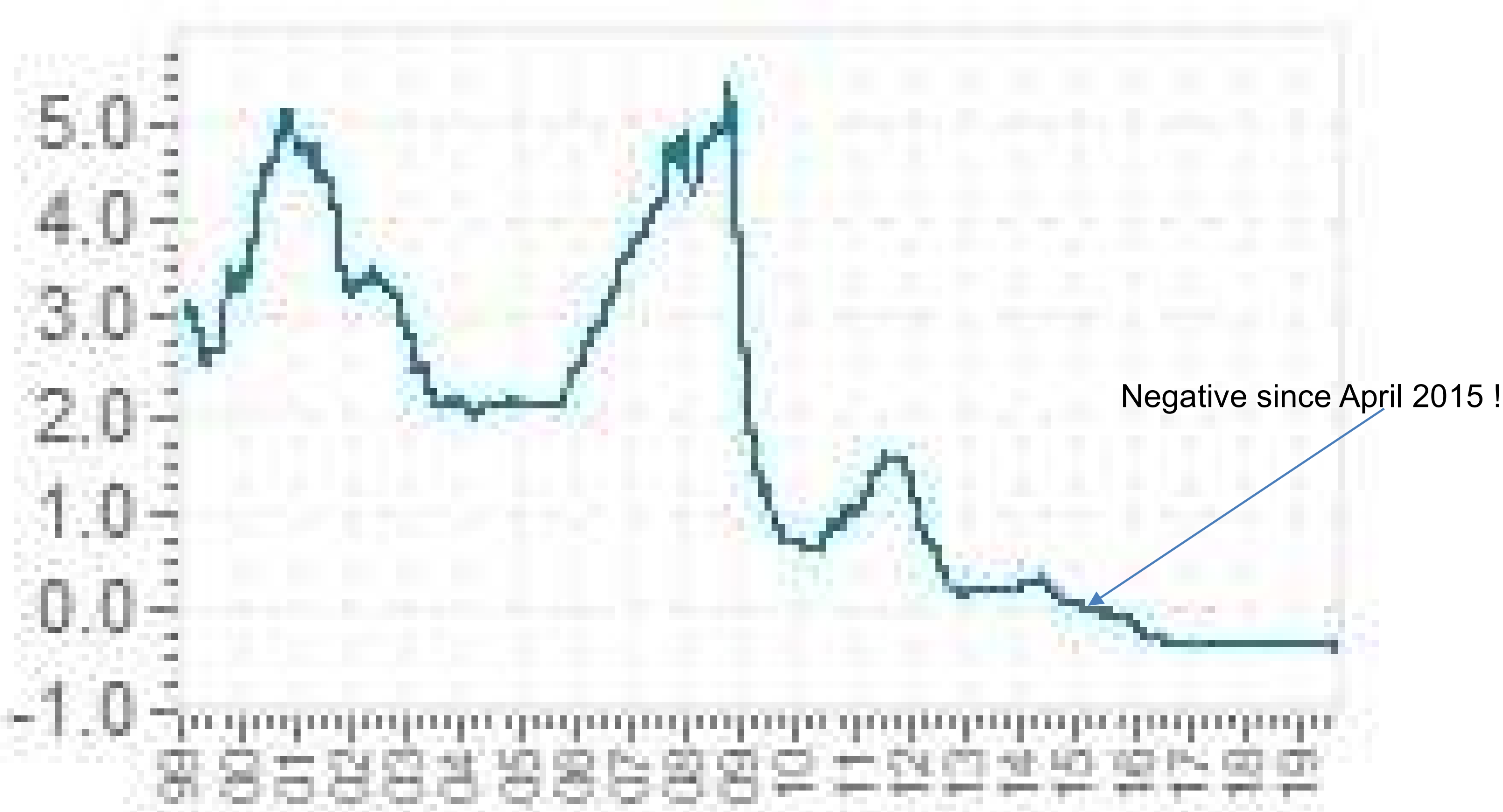
Important remark

- In all this course, we are going to assume that interest rates are positive.

- This assumption may seem straightforward.

- However, it is not always true, and isn't true at all in Europe since 4 years when negative interest rate started to by applied by the European Central Bank and them diseminate in euro money market and bond market.

Euribor 3 months : interbank short term interest rate



Source : https://fr.global-rates.com/taux-de-interets/euribor/taux-de-interets-euribor-3-mois.aspx

Current major interest rate benchmark

- Euribor (Euro Inter Bank Offered Rate) 3 months : -0,413 %

  A benchmark for interbank landing and borrowing (interbank mean the rate at witch a big european bank is lending money to another big european bank)

- Eonia (Euro Over Night Index Average) ; now replaced by €STR (Euro Short Term Rate) : -0,463 %.

Simple interest and compound interest

- Simple interests
- Example 1: the duration is an integer
- Let a loan of capital C = 1 000€ over a period of 6 months with a monthly rate r = 0.5%.
- What is the value of interest to pay ?
- Reply : $I = C \times r \times T = 1000 \times 0.5\% \times 6 = 30€$

Simple interest and compound interest

- Example 2 : Duration is not a whole number
- Let a loan of capital C = 1000 euros over a period of 6 months between January 1, 2005 and July 1, 2005 at the annual rate r = 5%. What is the amount of interest to pay ?
- Answer: The duration of the loan is 181 days, since 2005 is not leap. From where $T = 181/360 = 0.5027777 \ldots \approx 0.5$ year $I = 1000 \times (181/360) \times 5\% = 25{,}14$ euros
- Note: while we could have had the idea of using approximated 6 months = 1/2 year from which T = 0.5, we use for the calculation the exact value of T, that is, the 181/360 fraction
- 360 days is a market convention for Eurozone and US.

Simple interest and compound interest

- The simple interest method is used in the usual way for operations of less than one year. In this case: $I = CxTxr$
- In the case of transactions exceeding one year, the method of simple interests is not used. This is the interest method compounds that prevails.

The frequency of compounding

- So far, we have assumed that the interest was paid once at the end of each year; in other words, we have assumed the compounding on an annual basis or **annual compounding**.

- In practice, the interest payments may occur several times a year, e.g. on a monthly or semi-annual basis.

- This gives rise to different quoting conventions.

Interest rate quotes

- Nominal rate , or annual percentage rate, or stated rate is the rate quoted in the contract which indicates the amount of simple interest earned per year, i.e. it ignores the compounding effect.

- Periodic rate is the interest rate charged each compounding period

$$Periodic\ rate = \frac{Nominal\ rate}{k}$$

where *k* is the number of compounding periods per year

- Example: ABC Bank states a nominal rate for deposits of 6% with quarterly compounding, it means that you earn 6%/4 = 1.5% per quarter, which is the periodic rate.

Interest rate quotes

- Effective annual rate indicates the actual amount of interest that will be earned at the end of the year

$$EAR = (1 + periodic\ rate)^{k} - 1$$

Or:

$$EAR = \left(1 + \frac{Nominal\ rate}{k}\right)^{k} - 1$$

where *k* is the number of compounding periods per year

Interest rate quotes

- Practical work : Calculate the effective annual rate for a 10% investment with semiannual compounding.

Interest rate quotes

- Calculate the effective annual rate for a 10% investment with semiannual compounding.

$$EAR = \left(1 + \frac{0.1}{2}\right)^{2} - 1 = 10.25\%$$

- Should be indifferent between receiving 10.25% annual interest and receiving 10% interest, compounded semiannually.

Break Time ?

Frequency of compounding and EAR

- Practical work follows on !
- Suppose, as in the previous example, a nominal rate of 10%. What will happen to the EAR if the compounding is now a) quarterly; b) monthly?

Frequency of compounding and EAR

- Suppose, as in the previous example, a nominal rate of 10%. What will happen to the EAR if the compounding is now a) quarterly; b) monthly?

a) $EAR = (1 + \frac{0.1}{4})^{4} - 1 = 10.38\%$

b) $EAR = (1 + \frac{0.1}{12})^{12} - 1 = 10.47\%$

- EAR increases with the frequency of compounding, keeping nominal rate constant.

Converting a nominal rate to an EAR

- Practical work : your savings account terms are stated in the contract as « 4% nominal rate with monthly compounding ».
- What is the Effective Annual interest Rate ?
- What will your account balance be at the end of the first year if you have put $1,000 at the beginning of the year ?

Converting an APR to an EAR

$$1 + EAR = \left(1 + \frac{APR}{k}\right)^k$$

- Therefore, $EAR = \left(1 + \frac{0.04}{12}\right)^{12} - 1 = 0.0407$
- You effectively earn 4.07% per year.

- If you invest $1,000 at the beginning of the year, at the end of the year your account balance will increase to:

  1,000*(1+EAR) = 1,000*(1+0.0407) = $ 1,040.7

Converting an EAR to a nominal rate

- Practical work : suppose you invest $100 in a bank account. 1 year later it has grown to $110.
- What nominal rate does your bank provide if the interest was compounded quarterly ?