



Business Finance : course n°4

The Time Value of Money

BA 2nd Year - 01/10/2019

Three rules of time travel

Rule 1 Only values at the same point in time can be compared or combined.

Rule 2 To move a cash flow forward in time, you must compound it.

Future Value of a Cash Flow

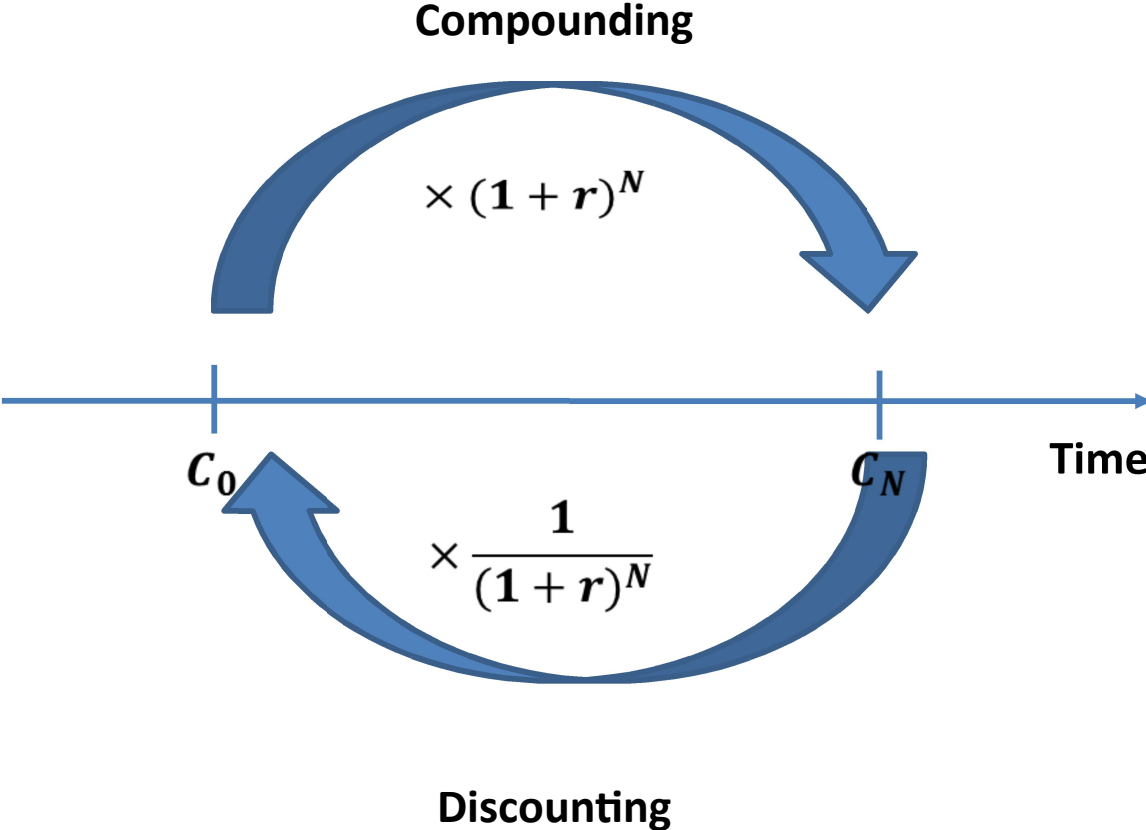
$$FV_n = C \times (1 + r)^n$$

Rule 3 To move a cash flow backward in time, you must discount it.

Present Value of a Cash Flow

$$PV = C \div (1 + r)^n = \frac{C}{(1 + r)^n}$$

Rules of time travel: illustration

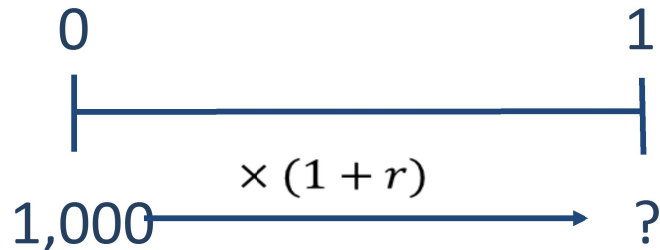


Moving cash flows forward in time: Future Value (FV) of a single cash flow

- Practical work : suppose you have \$1,000 today. Find the equivalent amount in 1 year if the current interest rate is 10% per year.

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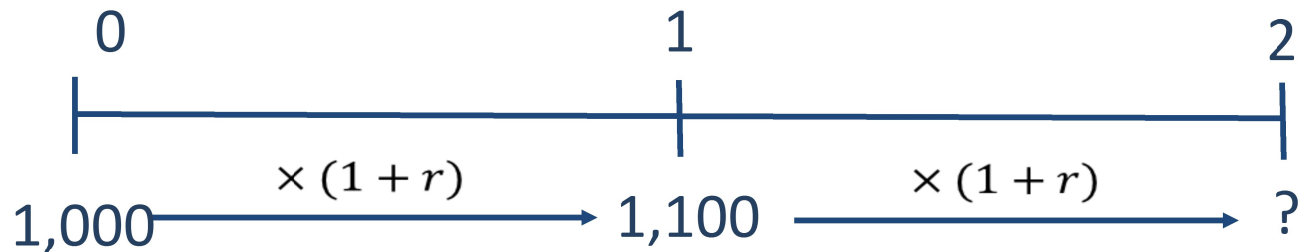
$$FV_1 = 1,000 \times (1 + 0.1) = \$1,100$$

Moving cash flows forward in time: Future Value (FV) of a single cash flow

- Practical work : suppose you have \$1,000 today. Find the equivalent amount in 2 years if the current interest rate is 10% per year.

Moving cash flows forward in time: Future Value (FV) of a single cash flow

- Suppose you have \$1,000 today. Find the equivalent amount in 2 years if the current interest rate is 10% per year.



$$FV_2 = 1,000 \times (1 + 0.1)^2 = \$1,210$$

Future value (FV) of a single cash flow: general formula

- FV of a cash flow C realized n years from now if the annual interest rate is r :

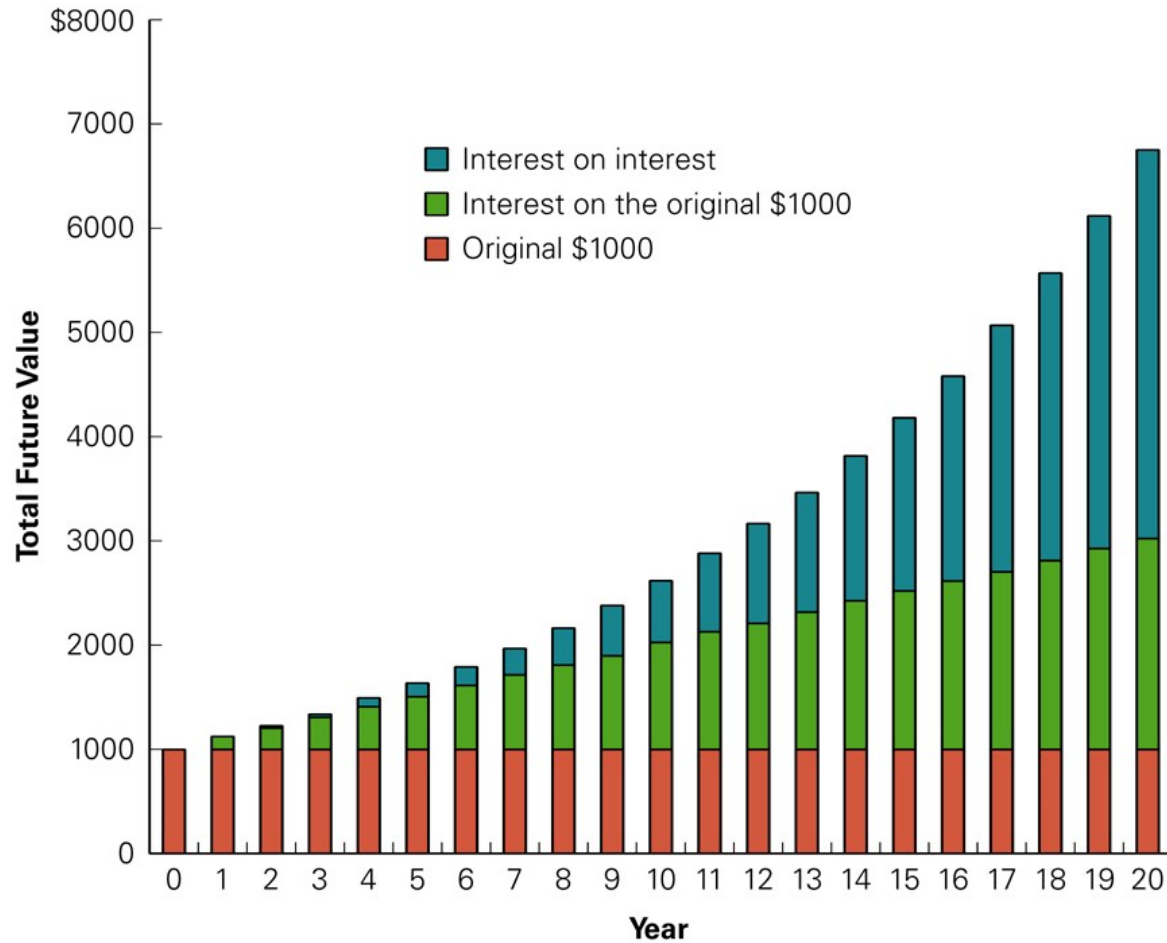
$$FV_n = C \times (1 + r) \times (1 + r) \times \cdots \times (1 + r)$$

$$\mathbf{FV_n = C \times (1 + r)^n}$$

Calculating the FV: compounding

- The process of moving a cash flow forward is also known as **compounding**.
- Note from the previous example that at the end of the first year your money grows by \$100, at the end of the 2nd year it grows by a larger amount, \$110.
- In the second year we earn interest on our original amount, \$1,000, and on the interest earned during the first year, \$100.
- The effect of earning « interest on interest » is known as **compound interest**.

Compound interest: intuition



Moving cash flows back in time: Present Value (PV) of a single cash flow

- Practical work : what is the value today of \$1,000 that you anticipate receiving in one year if the current interest rate is 10% per year ?

Moving cash flows back in time: Present Value (PV) of a single cash flow

- What is the value today of \$1,000 that you anticipate receiving in one year if the current interest rate is 10% per year?



$$PV = \frac{1,000}{1+0.1} = \$ 909.09$$

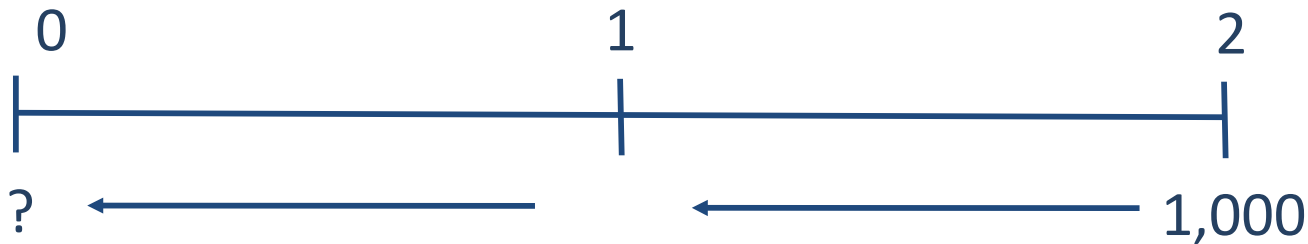
Break Time ?

Moving cash flows back in time: Present Value (PV) of a single cash flow

- Practical work : what is the value today of \$1,000 that you anticipate receiving in two years if the current interest rate is 10%?

Moving cash flows back in time: Present Value (PV) of a single cash flow

- What is the value today of \$1,000 that you anticipate receiving in two years if the current interest rate is 10%?



$$PV = \frac{1,000}{(1+0.1)^2} = \$ 826.45$$

Moving cash flows back in time: discounting

- The process of moving a cash flow back in time (finding the equivalent value today of a future cash flow) is known as **discounting**.
- The value of \$1 tomorrow is lower than the \$1 today => the value today reflects a discount.
 - Larger discount is applied for periods more distant in the future
 - As shown in the previous example, the today's equivalent of \$1,000 received in 2 years is lower than that of \$1,000 received in 1 year.

Present value (PV) of a single cash flow: general formula

- PV of a cash flow C realized n years from now if the interest rate is r :

$$PV = \frac{C}{(1 + r)^n}$$

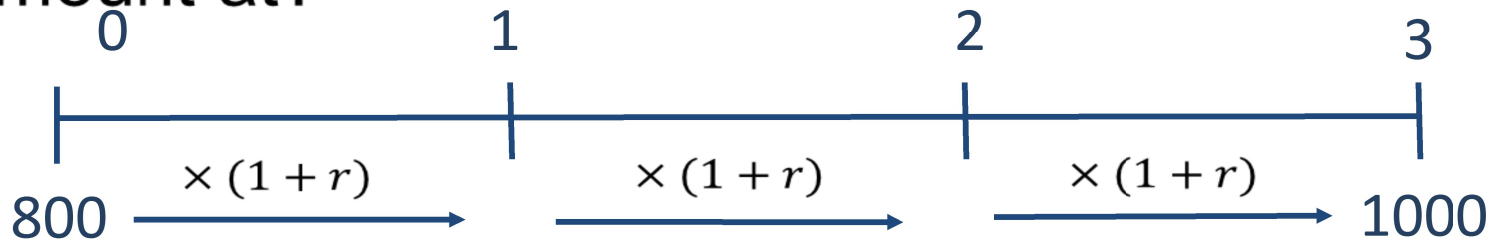
- The PV can be seen as today's equivalent of the amount received in the future.
- Alternatively, the PV can be considered the amount to be invested today at the current interest rate in order to receive the desired amount at the specified time.

Applying the rules of time travel

- Practical work : suppose you want your current savings of \$800 to grow to \$1,000 at the end of 3 years.
- What interest rate should you invest this amount at ?

Applying the rules of time travel

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$$FV_3 = C \cdot (1 + r)^3$$

$$r = \left(\frac{FV_3}{C} \right)^{1/3} - 1 = \left(\frac{1,000}{800} \right)^{\frac{1}{3}} - 1 = 7.72\%$$

Applying the rules of time travel

- Practical work : suppose you want to have 20 000\$ in 20 years, and on this period the stock market is expected to yield 8 % per year.
- What amount should you invest now ?

Applying the rules of time travel

- Practical work : suppose you want to have 20 000\$ in 20 years, and on this period the stock market is expected to yield 8 % per year.
- What amount should you invest now ?
- $20\,000\$/ (1+8\%)^{20} = 4\,290,96\$$